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A note on the Sorites paradox

Graham Priest [a]
[a] The University of Western Australia
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A NOTE ON THE SORITES PARADOX

Graham Priest

One of the principal aims of L. Zadeh and others in developing a theory of fuzzy sets was to provide a formal theory which could handle vagueness. Informal accounts of the Sorites paradox usually emphasise that the problem is essentially one of vagueness. (See e.g. S. Haack [1974] Ch. 6, p. 113 ff) It might be hoped therefore, that the theory of fuzzy sets allows for a formal resolution of the paradox. The purpose of this note is to show that it does.

Consider the following form of the paradox due to H. Wang. (See M. Dummett [1975])

<u>0 is a small number and if n is a small number, n+1 is.</u>

∴ All numbers are small.

The premiss seems to be true and the conclusion false. Yet the argument seems to be a perfectly valid instance of mathematical induction. What has gone wrong? The problem may be solved on the following two assumptions.

(i) Sentences may have varying degrees of truth. A truth value is a real number in the unit interval [01]. A sentence whose value is near 1 is true or pretty true. One whose value is near 0 is false or pretty false. (See e.g. G. Lakoff [1973].) Taking 'S' for the one place predicate 'is small', '$\underline{n}$' for the numeral corresponding to the number n, and $|A|$ for the truth value of A, we know that $|S\underline{n}| \simeq 1$ for n = 0, 1, 2 . . ., decreasing as n becomes larger until $|S\underline{n}| \simeq 0$ for sufficiently large n. For the sake of definiteness let us take $|S\underline{n}| = e^{-n/100}$. (So that $|S\underline{0}| = 1$, $|S\underline{100}| \simeq 0.37$.)

(ii) The truth values of compound sentences are truth functions of the truth values of their components. There are many suitable truth functions. The following, due to Lukasiewicz (see N. Rescher [1969] Ch. 2 §6) will suffice.

$|A \wedge B| = \text{Min } |A|, |B|$

$|A \rightarrow B| = 1 \text{ if } |A| \leq |B|$

$\qquad 1-(|A|-|B|) \text{ otherwise}$

$|\forall x A(x)| = \text{Min } \{|A(\underline{n})|; \text{ n a natural number}\}$

A is a logical consequence of B ($B \models A$) iff $|B| \leq |A|$.

Calculating under these assumptions, we obtain:

$$|S\underline{n} \rightarrow S\underline{n+1}| = 1-(e^{-n/100}-e^{-n-1/100})$$
$$= 1-e^{-n/100}\,(1-e^{-1/100})$$

The minimum value of this occurs when n = 0.

So $|\forall x\,(Sx \rightarrow Sx+1)| = e^{-1/100} \simeq 1$

But $|S\underline{o}| = 1$. Hence $|So \wedge \forall x\ (Sx \rightarrow Sx+1)| \simeq 1$

However, $|\forall xSx| = \text{Min}\ \{e^{-n/100};\ n \text{ a natural number}\} = 0.$

Hence, as we would expect, the premiss of the argument is pretty true and the conclusion false. Thus

$$S\underline{o} \wedge \forall x\ (Sx \rightarrow Sx+1) \not\models \forall xSx$$

Mathematical induction is shown to be an invalid form of argument when fuzzy properties are involved.

The University of Western Australia. Received March 1978

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