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## Australasian Journal of Philosophy

Publication details, including instructions for  
authors and subscription information:

<http://www.tandfonline.com/loi/rajp20>

### A note on the Sorites paradox

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Published online: 15 Sep 2006.

To cite this article: Graham Priest (1979) A note on the Sorites paradox,  
Australasian Journal of Philosophy, 57:1, 74-75, DOI: [10.1080/00048407912341061](https://doi.org/10.1080/00048407912341061)

To link to this article: <http://dx.doi.org/10.1080/00048407912341061>

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## A NOTE ON THE SORITES PARADOX

Graham Priest

One of the principal aims of L. Zadeh and others in developing a theory of fuzzy sets was to provide a formal theory which could handle vagueness. Informal accounts of the Sorites paradox usually emphasise that the problem is essentially one of vagueness. (See e.g. S. Haack [1974] Ch. 6, p. 113 ff) It might be hoped therefore, that the theory of fuzzy sets allows for a formal resolution of the paradox. The purpose of this note is to show that it does.

Consider the following form of the paradox due to H. Wang. (See M. Dummett [1975])

0 is a small number and if  $n$  is a small number,  $n+1$  is.

$\therefore$  All numbers are small.

The premiss seems to be true and the conclusion false. Yet the argument seems to be a perfectly valid instance of mathematical induction. What has gone wrong? The problem may be solved on the following two assumptions.

- (i) Sentences may have varying degrees of truth. A truth value is a real number in the unit interval [01]. A sentence whose value is near 1 is true or pretty true. One whose value is near 0 is false or pretty false. (See e.g. G. Lakoff [1973].) Taking 'S' for the one place predicate 'is small', ' $n$ ' for the numeral corresponding to the number  $n$ , and  $|A|$  for the truth value of A, we know that  $|Sn| \approx 1$  for  $n = 0, 1, 2 \dots$ , decreasing as  $n$  becomes larger until  $|Sn| \approx 0$  for sufficiently large  $n$ . For the sake of definiteness let us take  $|Sn| = e^{-n/100}$ . (So that  $|S0| = 1$ ,  $|S100| \approx 0.37$ .)
- (ii) The truth values of compound sentences are truth functions of the truth values of their components. There are many suitable truth functions. The following, due to Lukasiewicz (see N. Rescher [1969] Ch. 2 §6) will suffice.

$$|A \wedge B| = \text{Min } |A|, |B|$$

$$|A \rightarrow B| = 1 \text{ if } |A| \leq |B|$$

$$1 - (|A| - |B|) \text{ otherwise}$$

$$|\forall x A(x)| = \text{Min } \{ |A(\underline{n})|; n \text{ a natural number} \}$$

A is a logical consequence of B ( $B \models A$ ) iff  $|B| \leq |A|$ .

Calculating under these assumptions, we obtain:

$$\begin{aligned} |S\underline{n} \rightarrow S\underline{n+1}| &= 1 - (e^{-n/100} - e^{-n-1/100}) \\ &= 1 - e^{-n/100} (1 - e^{-1/100}) \end{aligned}$$

The minimum value of this occurs when  $n = 0$ .

So  $|\forall x (Sx \rightarrow Sx+1)| = e^{-1/100} \approx 1$

But  $|S_0| = 1$ . Hence  $|S_0 \wedge \forall x (Sx \rightarrow Sx+1)| \approx 1$

However,  $|\forall x Sx| = \text{Min} \{e^{-n/100}; n \text{ a natural number}\} = 0$ .

Hence, as we would expect, the premiss of the argument is pretty true and the conclusion false. Thus

$$S_0 \wedge \forall x (Sx \rightarrow Sx+1) \not\equiv \forall x Sx$$

Mathematical induction is shown to be an invalid form of argument when fuzzy properties are involved.

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Received March 1978

#### REFERENCES

- Dummett, M. [1975] 'Wang's Paradox' *Synthese* 30, pp. 301-324  
 Haack, S. [1974] *Deviant Logic* (Cambridge U.P.)  
 Lakoff, G. [1973] 'Hedges: A Study in Meaning Criteria and the Logic of Fuzzy Concepts' *Journal of Philosophical Logic* 2 pp. 458-508  
 Rescher, N. [1969] *Many-Valued Logic* (McGraw-Hill.)